

Self-starting stable coherent mode-locking in a two-section laser

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Coherent mode-locking (CML) uses self-induced transparency (SIT) soliton formation to achieve, in contrast to conventional schemes based on absorption saturation, the pulse durations below the limit allowed by the gain line width. Despite of the great promise it is difficult to realize it experimentally because a complicated setup is required. In all previous theoretical considerations CML is believed to be non-self-starting. In this article we show that if the cavity length is selected properly, a very stable (CML) regime can be realized in an elementary two-section ring-cavity geometry, and this regime is self-developing from the non-lasing state. The stability of the pulsed regime is the result of a dynamical stabilization mechanism arising due to finite-cavity-size effects.

I. INTRODUCTION

Development of ultrashort laser pulse sources with high repetition rates and peak power is an area of principal interest in optics. Such lasers have applications in a high-bit-rate optical communications, real time-monitoring of ultrafast processes in matter etc. A well-known method for generating high power ultrashort optical pulses is a passive mode-locking (PML) [1–6]. In order to achieve PML, a nonlinear saturable absorbing medium is placed into the laser cavity. In most of existing passively mode-locked lasers generation of ultrashort pulses arises due to the absorption/gain saturation in the gain and absorber sections [1, 7–9], with only few exceptions, such as in the case Kerr-lens mode-locking. Thus, in the most schemes the ultimate limit on the pulse duration τ_p is set by the medium polarization relaxation time T_2 , that is, $\tau_p \gtrsim T_2$. The interaction of the pulse with resonant gain and absorber media is not coherent in the sense that the medium polarization just follows the field and thus can be adiabatically eliminated [1, 7–14]. This is valid also in the case when the absorber enters the coherent regime (see below) whereas the gain medium is still in the usual regime of the gain saturation [15–19].

Another new way to achieve ultrashort pulse generation was proposed theoretically in [20, 21] [see Fig. 1(a)] where it was named “coherent mode-locking” (CML), or self-induced-transparency (SIT) mode-locking, as it is called sometimes [22, 23]. In this approach interaction of light with matter is so strong, that the medium polarization and inversion change significantly on the time scale of the pulse duration, and Rabi oscillations arise [24–26] (coherent regime). In this case, because the period of Rabi oscillations is not limited from below, the pulse duration can be significantly smaller then the medium coherence time, $\tau_p \ll T_2$, and the presence of the phase memory on the scale of T_2 changes the evolution of the pulse dramatically. Unlike the common passive mode-locking with a slow saturable absorber [1, 7, 27, 28], where the absorption is just saturated near the pulse cen-

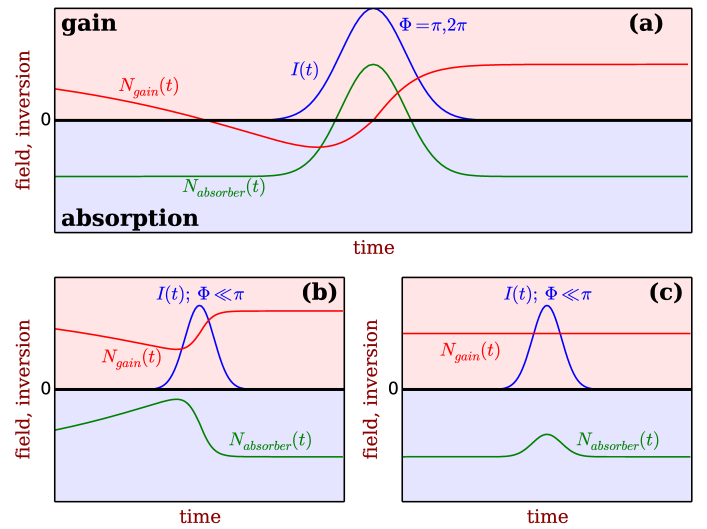


FIG. 1. The essential details of coherent mode-locking (CML) (sometimes referred to as “SIT mode-locking”). (a) The dependence of the field intensity $I(t)$ and population inversion $N_{\text{gain}}, N_{\text{absorber}}$ in the gain and absorber sections in the CML regime. The pulse area Φ [see Eq. (4)] is π in the gain section (a half of Rabi oscillation) and 2π in the absorber (a whole Rabi oscillation); in both sections the population inversion changes its sign. In the absorber, the population returns back on the pulse duration despite of much slower relaxation time T_{1a} of the absorber medium. The gain section is also switched from “amplifying” to “absorbing” state and then slowly, on the time scale of the population relaxation time T_{1g} , returns back. For comparison, in (b) passive mode-locking with a slow saturable absorber and gain saturation is demonstrated. In this case, both gain and absorber sections are only saturated, without onset of Rabi oscillations ($\Phi \ll \pi$). In this case, long tails of population inversion relaxing after the pulse passage appear. In (c), mode-locking with a fast absorber (such as Kerr lens) and no gain saturation is presented. Note some similarity of the absorber behavior in (a) and (c) despite of fundamentally different relaxation rates of the absorber [T_{1a} in (a) and instantaneous in (c)], which is the consequence of the coherent regime.

ter [see Fig. 1(b)], in the case of coherent interaction it is completely inverted, so that the population inversion crosses zero and changes its sign. As a result, such pulse propagates without losses in the absorber in the regime of self-induced-transparency (SIT) (2π pulse), thus forming a soliton. In the gain section the pulse takes all the energy from the medium (π pulse), making it highly absorbing (that is, again, the population inversion changes its sign), in contrast to common lasers schemes, where the population inversion in the gain section either does not change significantly [as in Fig. 1(c)], or changes relatively slightly, without crossing zero [as in Fig. 1(b)]. Because of this, such SIT-induced solitons are fundamentally different in their dynamics from the pulses appearing in the saturable schemes [27–29].

Contrary to the conventional passively mode-locked lasers with a saturable absorber, CML lasers can generate optical pulses with a duration much shorter than T_2 , i.e. with the spectrum exceeding the bandwidth of the gain medium. Moreover, it was predicted [20, 21, 30, 31] that pulse duration from CML lasers can approach the single optical cycle limit despite of narrow-band gain and absorber, even in the presence of inhomogeneous line broadening, dispersion, and the complex level structure [32].

Unfortunately, despite of the great promise, there was no experimental demonstration of CML in configuration proposed in [20, 21, 30, 31] up to now. However, generating of short pulses shorter than T_2 in mode-locked argon-ion laser with active mode-locker and in self-locked He-Ne laser was demonstrated experimentally in Refs. [33, 34] and [35], respectively. In our recent work [36] we have also shown experimentally a mode-locking regime with a pulse duration less than T_2 in the absorber. Also, in [22, 23] quantum cascade laser structures were proposed as candidates for experimental realization of CML regime. Theoretical study of CML performed in [20–23, 30, 31] was carried out for a laser with the gain and absorber implemented within the same sample, that is, as a “homogeneous mixture” of the amplifying and absorbing media. Such proposals have, however, some important disadvantages. First, the pulsed regime can not develop spontaneously from a non-lasing state. Namely, to ensure the stability of the pulse one have to suppress the background fluctuations far away from it, thus automatically making the non-lasing steady state stable and the whole laser non-self-starting. That is, to initiate a soliton, one needs a seed pump pulse injected to the laser. The necessity to make CML lasers non-self-starting follows thus from a solitonic character of the coherent mode-locking. In contrast, many non-CML mode-locking lasers can indeed stably self-start even if the absorber (but not the amplifier) works in the coherent regime [16, 19]. Nevertheless the problem of self-starting is also actual for other existing fast modelocking schemes such as Kerr-lens mode-locking [5]. The second important drawback is the necessity of a “homogeneous mixture” of the amplifier and absorber assumed in the works on CML up to now, which, although ensures a solitonic character of the

mode-locking, makes its practical implementation rather difficult.

In this article, we consider a simple scheme of CML-based modelocking with the amplifying and absorbing media being well separated spatially, forming rather usual two-section geometry. A possibility of the CML in this case was shortly reported by us in [37]. In the present article we focus on the problem of self-starting of CML regime in such geometry and analyze in details characteristic behaviour of the system. We demonstrate that if the cavity length is selected properly, we can cross the point where the non-lasing state is becoming unstable, nevertheless obtaining good fundamental CML regime. That is, no need of a seed pulse is necessary anymore, in contrast to previous considerations [20–23, 30, 31]. The resulting pulsed attractor is stabilized globally due to finite-size effects in the cavity. We focus our attention on the gaseous media with T_2 in the range of nanoseconds, so that we have no need to approach single-cycle limit to overcome T_2 , which significantly simplifies the required model. Nevertheless, this scheme is very attractive as a source of picosecond pulses with high power and high repetition rate > 1 GHz.

II. THE MODEL

The ring-cavity configuration considered in this article is shown in Fig. 2a. Between the mirrors, only one of which is assumed partially reflecting with the reflection coefficient R , and the others are “ideal” for simplicity, the gain and the absorber sections are placed. Both sections consist of resonant nonlinear medium, tuned to the same frequency. The coupling to the field, namely the dipole moment of active “atoms” is different for both sections. The media and field are described in the two-level and slowly-varying envelope approximations respectively [26, 38, 39]:

$$\partial_t P(z, t) = -\frac{P(z, t)}{T_2(z)} + \frac{d_{12}(z)}{2\hbar} \Delta\rho(z, t) A(z, t), \quad (1)$$

$$\partial_t \Delta\rho(z, t) = -\frac{\Delta\rho(z, t) - \Delta\rho_0(z)}{T_1(z)} - \frac{d_{12}(z)}{2\hbar} F(z, t), \quad (2)$$

$$\partial_t A(z, t) - c\partial_z A(z, t) = 4\pi\omega_{12}d_{12}(z)N_0(z)P(z, t). \quad (3)$$

Here $P(z, t)$ is the slowly-varying envelope of the non-diagonal element of the density matrix describing the two-level atom in the absorber (for $0 < z < L_a$) and gain (for $L_a < z < L \equiv L_a + L_g$) sections in rotating wave approximation; $\Delta\rho(z, t)$ is the difference of diagonal elements of the density matrix (population difference per single atom) in the gain and absorber sections, $A(z, t)$ is the slowly varying field amplitude in the gain and absorber sections. $\omega_{12} = 2\pi c/\lambda_{12}$ is the transition frequency, c is the speed of light in vacuum, and $F(z, t) = A(z, t)P(z, t)$; The other parameters, and their values in numerical simulations are given in the Table I and are characteristic

for gases [40]. The cavity length is $L = 30$ cm (corresponding to round-trip time $\tau = 1$ ns), and the mirror reflectivity is $R = 0.8$. The equations (1)-(3) are highly used in different physical situations describing resonant behavior of nonlinear media [24–26, 41–45].

We remark that, because we assume the gaseous media with T_2 in the range of nanoseconds, we do not need to approach single-cycle pulse limit to achieve the coherent (SIT) regime. In our case the pulse duration will be > 1 ps, thus, the dispersion of the cavity plays only the minor role. Besides, because the ratio of Rabi frequency to ω_{12} is $\ll 10^{-3}$ in our simulations, the two-level approximation can be assumed as very good one.

Important quantity characterizing the pulse propagation dynamics in two level atoms is the pulse area Φ :

$$\Phi(z, t) = \frac{d_{1,2}}{\hbar} \int_{-\infty}^{\infty} A(z, \tau) d\tau \approx \frac{d_{1,2}}{\hbar} \int_{t-4\tau_p}^{t+4\tau_p} A(z, \tau) d\tau, \quad (4)$$

where τ_p is the pulse duration; that is, we integrate in the region around the pulse to get practically useful measure in the case when other pulses are present.

III. RESULTS OF NUMERICAL SIMULATIONS

According to the original idea of the coherent soliton mode locking [20–23, 30, 31], the dipole moments of the gain is two times smaller than in the absorber, which ensures, together with the “mixed” character of the medium, that fluctuations in the near of the non-lasing state decay. Such situation can be described from the point of view of nonlinear dynamics [46] in the following way: the stable pulsed regime exists and is well separated in the phase space (see inset to Fig. 1c) from the non-lasing steady state, which is also stable. That is, the system is attracted to the regime, in which vicinity it is located initially. Other regimes may exist but they are irrelevant for our consideration. To achieve a stable pulsed regime in this situation one needs a large-intensity “perturbation”, that is, a seed pulse, which makes the scheme more complicated from the practical point of view.

Although in the infinite-length, “mixed” medium the soliton will be shortened until it reaches single-cycle regime, in the cavity geometry with the limited cavity length the pulse duration achieves its stationary value τ_p , which is determined by the cavity parameters. The order of magnitude of τ_p , e.g. in absorber, can be obtained from the requirement $\Phi \approx 2\pi$ leading to $A\tau_p \approx 2\pi\hbar/d$. Using also the energy balance in the gain section, that is, assuming that the energy of the pulse produced by the atoms $w \approx \hbar\omega_{12}L_gN_g$ is fully transferred to the field, that is, $w \approx A^2\tau_p/8\pi c$, we will finally obtain:

$$\tau_p \approx \pi\hbar/(d_a^2\omega_{12}N_gL_g), \quad (5)$$

which gives for the parameters we use the order of tens picoseconds. We note that Eq. (5) gives only very raw

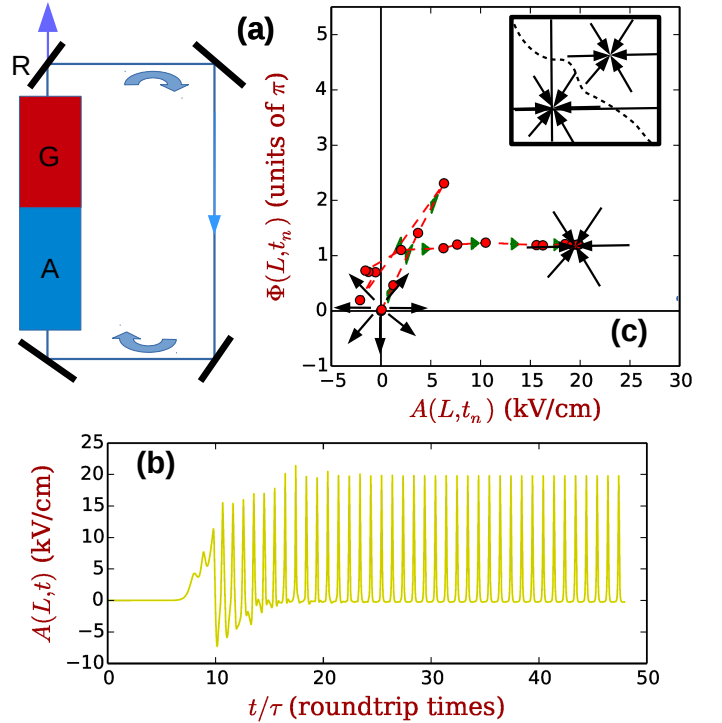


FIG. 2. (a) The scheme of the ring-cavity laser with an absorber (A) and gain (G) sections. (b) The output field $A(L, t)$ evolution in dependence on time during a buildup of a stable pulsed regime from a vicinity of the non-lasing state. (c) The map in a plane of the pulse area $\Phi(L, t_n)$ and field $A(L, t_n)$ in the time moments $t_n = t_0 + n\tau$, where $n = 1, 2, \dots$ and t_0 is a constant chosen to align t_n to the pulse maxima at large t_n . Black arrows show the attracting or repelling character of every steady-state. Green arrows show the direction of the system evolution. In the inset the corresponding dynamical picture for the configuration from [20] is shown. The laser parameters are given in Table I.

approximation of the pulse duration. Better approximation is possible using the equations given in [20, 23].

We assume the lengths of the gain and absorber sections L_g and L_a , larger than the pulse length $c\tau_p$, $L_a, L_g > c\tau_p$. Importantly, we also assume that the cavity is small enough to prevent the population difference to relax to their equilibrium states, that is $\tau \ll T_{1g}, T_{1a}$. The last condition is crucial and makes the pulse propagating in the cavity essentially non-solitonic as we will see later. The important parameter G_0 determining the gain-loss balance for low-intensity fluctuations and thus the stability of non-lasing state is defined as:

$$G_0 = \ln \left[R \exp \left\{ \int_0^L \alpha(z) dz \right\} \right], \quad (6)$$

where $\alpha(z) = -2\pi d_{12}^2(z)\omega_{12}N_0(z)\Delta\rho(z)T_2(z)/\hbar c$ is the gain/loss coefficient for low-intensity fluctuations. We chose N_a and N_g (see Table 1) in such a way that $G_0 > 0$, facilitating growth of low-frequency fluctuations in the vicinity of nonlasing state and thus making it unstable.

TABLE I. Parameter values used in the numerical simulations

Parameter	Gain ($L_a < z < L$)	Absorber ($0 < z < L_a$)
central transition wavelength	$\lambda_{12} = 0.6 \mu m$	$\lambda_{12} = 0.6 \mu m$
length of the medium	$L_g = 15 \text{ cm}$	$L_a = 15 \text{ cm}$
concentration of two-level atoms, $N_0(z) = \dots$	$N_{0g} = 4.0 \cdot 10^{13} \text{ cm}^{-3}$	$N_{0a} = 0.8 \cdot 10^{13} \text{ cm}^{-3}$
transition dipole moment, $d_{12}(z) = \dots$	$d_{12g} = 0.5 \text{ Debye}$	$d_{12a} = 1 \text{ Debye}$
population difference at equilibrium, $\Delta\rho_0(z) = \dots$	$\Delta\rho_{0g} = -1$	$\Delta\rho_{0a} = 1$
population difference relaxation time, $T_1(z) = \dots$	$T_{1g} = 6 \text{ ns}$	$T_{1a} = 6 \text{ ns}$
polarization relaxation time, $T_2(z) = \dots$	$T_{2g} = 0.5 \text{ ns}$	$T_{2a} = 0.5 \text{ ns}$

The parameters for our numerical simulations are shown in Table I. One can see that $G_0 \approx 9.0 > 0$, that is, the non-lasing state is strongly unstable. We start the simulations from its vicinity, assuming the field close to zero, the population differences at their equilibrium values and the polarization to be zero, and track the evolution of the field in the cavity with time. An exemplary trajectory is shown in Fig. 2b, where we have started from the a short probe pulse of the the duration $\tau_{p0} = 100 \text{ ps}$ and initial pulse area is $\Phi_0 = 10^{-7}\pi$.

After a short but irregular transient process up to $t/\tau \sim 20$ a stable regime with the pulse duration $\tau_p = 80 \text{ ps}$ is born, which is significantly smaller that the relaxation time T_2 , thus corroborating the coherent character of the mode-locking regime in this case.

We tested numerically the achievability and stability of the pulsed regime, performing many simulations starting initial conditions randomly selected every time in large range of pulse shapes, with or without initial noise. All the trajectories were attracted to the single stable steady-state identical to the shown in Fig. 2b,c up to a time shift and sign of the output field.

The details of the resulting steady-state regime are shown in Fig. 3, where the pulse shapes in the absorber Fig. 3(a) and amplifier Fig. 3(b) sections are shown, together with the pulse area Φ in the cavity Fig. 3(c). As on can see, the pulse area in absorber, initially (at $z = 0$) being reduced to around 1.9π by the losses at the mirror, approaches quickly it's stable value 2π . The population difference is positive before the pulse, becomes negative at its leading edge, and at the trailing edge is restored to the initial value demonstrating dynamics which is typical for 2π -pulses. In addition, despite of the lossy character of the absorbing medium, the pulse energy does not decreases upon the propagation in the absorber, as it is shown in Fig. 3(d), which is also typical for 2π pulses. In the gain section, the pulse area is close to π , as seen from Fig. 3(c). Typically for such pulses, the population inversion changes its sign after the pulse propagation. Also, as seen from Fig. 3(d), the pulse “eats” the energy from the gain medium, so its full energy increases. Then, the energy obtained in the gain section is radiated thought the mirror at $z = L$.

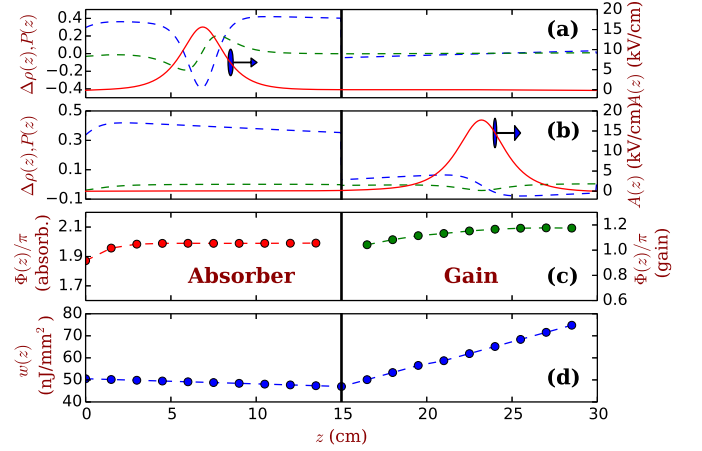


FIG. 3. Pulse shapes in the amplifier (a) and absorber (b) sections in the stationary regime. The field amplitude $A(z)$ (red line) as well as the polarization $P_s(z)$ (green dashed line) and population difference $\Delta\rho(z)$ (blue dashed line) are shown for the parameters in Table I. Note the difference in sign in comparison to N_{gain} , $N_{absorber}$ in Fig. 1. In (c) and (d), the pulse area Φ (c) and the pulse energy flux w (d) in the gain and absorber sections are shown.

Importantly, the population difference before the pulse is far from its equilibrium value, and rather close to zero (although remains negative), that is, the medium is in slightly amplifying regime before the pulse. After the pulse, the gain medium is switched to slightly absorbing regime. As the relaxation time $T_{1g} \gg \tau$, the gain section has no time to achieve its stationary value after the pulse. In contrast, the population difference in the absorber remains comparable to its equilibrium value. As a result, the gain-loss balance for the small perturbations becomes negative, protecting the pulse against destroying. Namely, G_0 in the such pulsed regime (obtained by integrating everywhere outside of the pulse in Eq. 6) becomes negative, in particular for parameters in Fig. 2 $G_0 \approx -1.0$.

This dynamics is rather typical for wide range of the parameters as far as the conditions $L_a, L_g > c\tau_p$ and $L \ll cT_{1g}, cT_{1a}$, $G_0 > 0$ (and is not very large, $G_0 \lesssim 20$) are

satisfied. For instance, in Fig. 4 the dependence of pulse duration and the peak intensity on the dipole moment of absorber d_{12a} is shown, assuming the other parameters except N_{0a} being unchanged, the later is scaled such that G_0 remains constant. The pulse durations are far from the single-cycle limit but nevertheless are significantly smaller than the phase coherence time.

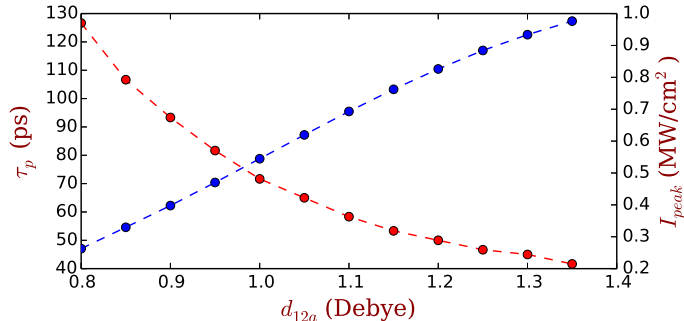


FIG. 4. The pulse duration τ_p (red line) and the peak intensity I_{peak} (blue line) in dependence on the d_{12a} in the stationary regime, changing the concentration in absorber section in such a way that G_0 remains the same, and keeping all other parameters as in Table I

Now let us briefly consider what happens if we cross these cavity length limits in various directions. If L_g approaches cT_{1g} , the population difference is able to recover before the pulse returns. In the other words, now more than one such pulse can propagate inside the cavity. In this situation, according to our simulations, the number of pulses in the cavity is indeed larger than one. The pulses typically interact with each other, leading to a complicated dynamics, so that the perfect mode-locking does not take place anymore. An example of such regime is presented in Fig. 4 (blue line) for $L = 50$ cm. In the case when L decreases and becomes

comparable with τ_p , the soliton-like structure can not anymore exist, and either (a spatially inhomogeneous) steady-state, or periodic regime with the low modulations depth appears, as shown in Fig. 4 (green and red lines) for $L = 18$ cm and $L = 20$ cm, correspondingly.

IV. DISCUSSION AND CONCLUSION

In conclusion, we have shown numerically that a stable self-starting CML allowing pulse durations significantly smaller than T_2 can be obtained in a laser with absorber and gain sections separated in space. No seed pulse and no complicated, difficult to realize artificial geometry considered in previous works on CML is needed. Rather trivial ring-cavity two-section laser with properly selected parameters makes the stable mode-locked regime being automatically self-started from the non-lasing state. This is a significant advantages over the previous approaches to CML which were all non-self-starting. To achieve self-starting the non-lasing state must be unstable and, in ad-

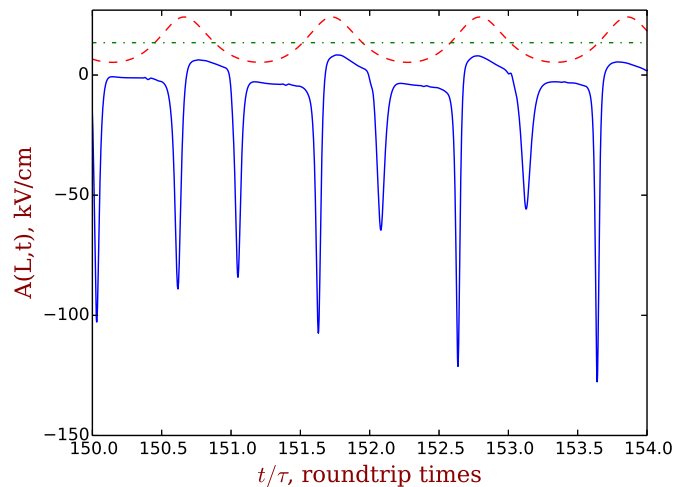


FIG. 5. The field dynamics at the laser output $A(L, t)$ in dependence on time for different cavity lengths $L = 50$ cm (blue solid line), $L = 20$ cm (red dashed line), $L = 18$ cm (green dash-dotted line). The other parameters are as in Fig. 1.

dition, the cavity round-trip time τ must be chosen to be significantly smaller than the population difference relaxation time T_{1g} but still significantly larger than the pulse duration. In this case, during the onset of generation process, initial fluctuations start to grow until the laser enters significantly nonlinear and even coherent regime with $\Phi \sim 1$. This initiates a chaotic behaviour with, typically, a train of several irregular pulses propagating in the cavity. This is a point where our stabilization mechanism starts to work. After a passage of the strongest pulse the population inversion starts to restore on the time scale of T_{1g} . If the cavity lengths is selected properly, this pulse makes the full roundtrip and returns back just at the point when the restoration is (almost) completed, whereas the other pulses “fill” smaller gain and thus are depleted. In this way, the mode-locking regime with only one pulse per roundtrip time is stabilized.

In the other words, the mechanism considered here stabilizes the fundamental modelocking regime against the multiple pulses per roundtrip time. In this respect, we are also free from the necessity to consider development of the fluctuation dynamics at the lasing threshold, as it is typically made for self-starting modelocking schemes in the cases when the fundamental modelocking is developing from the non-lasing steady state [16, 47–56]. In the same way it is clear that the mechanism described here gives the modelocking with the probability one. We remark also that, because of the instability of the off state, the regime considered here also can not be described as a temporal cavity soliton [57].

We considered here a case of a diluted gas with typical T_2 in the range of nanoseconds, which makes the description of the problem rather simple even if the pulse duration $\tau \ll T_2$. In our case τ is in the range of tens

of picoseconds. In particular, dispersion of the cavity elements play no role and can be neglected. Also, because the Rabi frequency is also very small in comparison to ω_{12} , the two-level approximation considered here is also rather good.

Nevertheless, even in the present case, far from the single-cycle limit and with rather low density of the active atoms, CML-based scheme demonstrates impressive pulse repetition rate in the range of GHz and high power density. We remark that our preliminary simulations show the possibility of such stable CML regime for large range of parameters (cavity length, dipole moments etc),

for instance, typical for semiconductor lasers. Nevertheless, to describe such system more comprehensive modeling is needed which will be subject of our future studies.

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